FORMULĖS testui T1 – Integralai

Matematika 2

Elementariųjų funkcijų integralų lentelė

1. $\int du = u + C$

2. $\int u^{\alpha} du = \frac{u^{\alpha+1}}{\alpha+1} + C, \ \alpha \neq -1$ 2a. $\int \frac{du}{\sqrt{u}} = 2\sqrt{u} + C$

3. $\int \frac{du}{u} = \ln|u| + C$ 2b. $\int \frac{du}{u^2} = -\frac{1}{u} + C$

4. $\int e^u du = e^u + C$ 5. $\int a^u du = \frac{a^u}{\ln a} + C$

6. $\int \cos u du = \sin u + C$ 7. $\int \sin u du = -\cos u + C$

8. $\int \frac{du}{\cos^2 u} = \operatorname{tg} u + C$ 9. $\int \frac{du}{\sin^2 u} = -\operatorname{ctg} u + C$

10. $\int \frac{du}{\sqrt{1-u^2}} = \arcsin u + C$ 11. $\int \frac{du}{1+u^2} = \operatorname{arctg} u + C$

Kai kurie dažnai pasitaikantys integralai

12. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin(\frac{u}{a}) + C$ 13. $\int \frac{du}{a^2+u^2} = \frac{1}{a}\operatorname{arctg}(\frac{u}{a}) + C$

14. $\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$

15. $\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

16. $\int \sqrt{a^2-u^2}\, du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\arcsin(\frac{u}{a}) + C$

17. $\int \sqrt{u^2 \pm a^2}\, du = \frac{u}{2}\sqrt{u^2 \pm a^2} \pm \frac{a^2}{2}\ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

18. $\int \frac{du}{\sin u} = \ln\left|\operatorname{tg}(\frac{u}{2})\right| + C$ 19. $\int \frac{du}{\cos u} = \ln\left|\operatorname{tg}\left(\frac{\pi}{4} + \frac{u}{2}\right)\right| + C$

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Trigonometriniai keitiniai

$\operatorname{tg}\frac{x}{2} = t, \ dx = \frac{2dt}{1+t^2}, \ \sin x = \frac{2t}{1+t^2}, \ \cos x = \frac{1-t^2}{1+t^2}$

$\operatorname{tg} x = t, \ dx = \frac{dt}{1+t^2}, \ \sin x = \frac{t}{\sqrt{1+t^2}}, \ \cos x = \frac{1}{\sqrt{1+t^2}}$

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$$\int_0^{\frac{\pi}{2}} \sin^n x\, dx = \int_0^{\frac{\pi}{2}} \cos^n x\, dx = \begin{cases} \frac{(n-1)!!}{n!!}, \text{ kai } n \text{ - nelyginis sk.,} \\ \frac{(n-1)!!}{n!!}\cdot\frac{\pi}{2}, \text{ kai } n \text{ - lyginis sk., } n \in N. \end{cases}$$

Elementariųjų funkcijų išvestinių lentelė

1. $C' = 0$ 2. $x' = 1$ 3. $(u^{\alpha})' = \alpha u^{\alpha-1}\cdot u'$

3a. $\left(\sqrt{u}\right)' = \frac{1}{2\sqrt{u}}\cdot u'$ 3b. $\left(\frac{1}{u}\right)' = -\frac{1}{u^2}\cdot u'$

4. $(\ln u)' = \frac{1}{u}\cdot u'$ 5. $(\log_a u)' = \frac{1}{u\ln a}\cdot u'$

6. $(e^u)' = e^u \cdot u'$ 7. $(a^u)' = a^u \ln a \cdot u'$

8. $(\sin u)' = \cos u \cdot u'$ 9. $(\cos u)' = -\sin u \cdot u'$

10. $(\operatorname{tg} u)' = \frac{1}{\cos^2 u}\cdot u'$ 11. $(\operatorname{ctg} u)' = -\frac{1}{\sin^2 u}\cdot u'$

12. $(\arcsin u)' = \frac{1}{\sqrt{1-u^2}}\cdot u'$ 13. $(\arccos u)' = -\frac{1}{\sqrt{1-u^2}}\cdot u'$

14. $(\operatorname{arctg} u)' = \frac{1}{1+u^2}\cdot u'$ 15. $(\operatorname{arcctg} u)' = -\frac{1}{1+u^2}\cdot u'$

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Trigonometrijos formulės

$\sin^2 x + \cos^2 x = 1$

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$1 + \operatorname{tg}^2 x = \frac{1}{\cos^2 x}$

$1 + \operatorname{ctg}^2 x = \frac{1}{\sin^2 x}$

$\sin\alpha\cdot\cos\beta = \frac{1}{2}\left(\sin(\alpha-\beta) + \sin(\alpha+\beta)\right)$

$\cos\alpha\cdot\cos\beta = \frac{1}{2}\left(\cos(\alpha-\beta) + \cos(\alpha+\beta)\right)$

$\sin\alpha\cdot\sin\beta = \frac{1}{2}\left(\cos(\alpha-\beta) - \cos(\alpha+\beta)\right)$

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Trigonometrinių funkcijų reikšmės

$\cos(-x) = \cos x,$ $\sin(-x) = -\sin x, \ \operatorname{tg}(-x) = -\operatorname{tg} x,$

$\operatorname{ctg}(-x) = -\operatorname{ctg} x, \ \operatorname{arctg}(-x) = -\operatorname{arctg} x$

$\operatorname{arctg}(-\infty) = -\frac{\pi}{2}, \quad \operatorname{arctg}(+\infty) = \frac{\pi}{2}$

α	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	– 1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	– 1	0	1
$\operatorname{tg}\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	–	0	–	0
$\operatorname{ctg}\alpha$	–	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	–	0	–

Kreivinės trapecijos ploto, kreivės lanko ilgio ir sukinio tūrio formulės

Kreivės lygtis	Plotas po kreive	Kreivės lanko ilgis	Sukinių tūriai
$y = f(x), \ x \in [a;b]$	$S = \int_a^b f(x)dx$	$L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$	$V_x = \pi\int_a^b (f(x))^2 dx, \quad V_y = 2\pi\int_a^b x\cdot f(x)dx$
$x = g(y), \ y \in [c;d]$	$S = \int_c^d g(y)dy$	$L = \int_c^d \sqrt{1 + (g'(y))^2}\, dy$	$V_x = 2\pi\int_c^d y\cdot g(y)dy, \quad V_y = \pi\int_c^d (g(y))^2 dy$
$\begin{cases} x = x(t), \\ y = y(t), \ t \in [t_1;t_2] \end{cases}$	$S = \int_{t_1}^{t_2} y(t)\cdot x'(t)dt$	$L = \int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$	