

Užduotis.

Apskaičiuokite apskritimo, duoto parametrinėmis lygtimis $\begin{cases} x = 3\cos(t) \\ y = 3\sin(t) \end{cases}$, liestinę, kai $t = \frac{\pi}{4}$.

Nubraižykite grafiką.

Sprendimas.

Liestinės lygtis:

$$y - y_0 = y'(x_0)(x - x_0), \text{ kai } y = f(x).$$

$$y - y_0 = y'_x(t_0)(x - x_0), \text{ kai } \begin{cases} x = f(t) \\ y = g(t) \end{cases}.$$

Randame tašką $M_0(x_0; y_0)$:

$$x_0 = 3\cos\left(\frac{\pi}{4}\right) = \frac{3\sqrt{2}}{2}$$

$$y = 3\sin\left(\frac{\pi}{4}\right) = \frac{3\sqrt{2}}{2}$$

Taigi gavom $M_0\left(\frac{3\sqrt{2}}{2}; \frac{3\sqrt{2}}{2}\right)$.

$$\text{Ieškom } y'_x = \frac{y'_t}{x'_t} = \frac{3\cos(t)}{-3\sin(t)} = -\cot g(t)$$

$$y'_x\left(\frac{\pi}{4}\right) = -\cot g\left(\frac{\pi}{4}\right) = -1.$$

Taigi liestinė lygi:

$$y - y_0 = y'_x(t_0)(x - x_0)$$

$$y - \frac{3\sqrt{2}}{2} = -\left(x - \frac{3\sqrt{2}}{2}\right)$$

$$y = -x + 2 \cdot \frac{3\sqrt{2}}{2} = 3\sqrt{2} - x.$$

