3 PASKAITA (papildomai)
Funkcijų, kurių išraiškoje yra kvadratinis trinaris, integravimas

Nagrinėkime integralą $I_1 = \int \frac{dx}{ax^2+bx+c}$.

Vardiklyje išskiriame dvinario kvadratą $ax^2+bx+c = a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right) = a\left(\left(x+\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2+\frac{c}{a}\right) = a\left(\left(x+\frac{b}{2a}\right)^2 \pm k^2\right)$ ($\left(\frac{b}{2a}\right)^2+\frac{c}{a}$ pažymėjome *k*).

Tada parinkę keitinį $t = x+\frac{b}{2a}$, $dx = dt$, gausime:

$$I_1 = \frac{1}{a}\int \frac{dt}{t^2 \pm k^2}.$$

1) $\int \frac{dx}{x^2+2x+3} = \int \frac{dx}{(x+1)^2+2} = \int \frac{dx}{(x+1)^2+\left(\sqrt{2}\right)^2} = \begin{bmatrix} t = x+1 \\ dt = dx \end{bmatrix} = \int \frac{dt}{t^2+\left(\sqrt{2}\right)^2} = \frac{1}{\sqrt{2}} arctg\frac{t}{\sqrt{2}}+C = \frac{1}{\sqrt{2}} arctg\frac{x+1}{\sqrt{2}}+C$;

2) $\int \frac{dx}{x^2-2x+5} = \int \frac{dx}{(x-1)^2+4} = \int \frac{dx}{(x-1)^2+2^2} = \begin{bmatrix} t = x-1 \\ dt = dx \end{bmatrix} = \int \frac{dt}{t^2+2^2} = \frac{1}{2} arctg\frac{t}{2}+C = \frac{1}{2} arctg\frac{x-1}{2}+C$.

Jei

$$I_2 = \int \frac{Mx+N}{ax^2+bx+c},$$

tada $I_2 = \int \frac{\frac{M}{2a}(2ax+b)+\left(N-\frac{Mb}{2a}\right)}{ax^2+bx+c}dx = \frac{M}{2a}\int \frac{d\left(ax^2+bx+c\right)}{ax^2+bx+c}+\left(N-\frac{Mb}{2a}\right)\int \frac{dx}{ax^2+bx+c} = \frac{M}{2a}\ln\left|ax^2+bx+c\right|+\left(N-\frac{Mb}{2a}\right)\cdot I_1+C$

3) $\int \frac{4x-3}{x^2+3x+4}dx = \int \frac{2(2x+3)-3-6}{x^2+3x+4}dx = 2\int \frac{2x+3}{x^2+3x+4}dx - 9\int \frac{dx}{x^2+3x+4} = 2\int \frac{d\left(x^2+3x+4\right)}{x^2+3x+4} - 9\int \frac{dx}{(x+1{,}5)^2+\frac{7}{4}} =$

$= 2\ln\left|x^2+3x+4\right| - \frac{9}{\frac{\sqrt{7}}{2}} arctg\frac{x+1{,}5}{\frac{\sqrt{7}}{2}}+C.$

4) $\int \frac{1-2x}{x^2+2x+10}dx = C-\ln\left|x^2+2x+10\right|+arctg\frac{x+1}{3}.$

Jei turime tokio tipo integralą:

$$I_3 = \int \frac{dx}{\sqrt{ax^2+bx+c}},$$

tada jį pertvarkome kaip I_1, turėsime $\int \frac{dx}{\sqrt{t^2 \pm k^2}}$ $(a>0)$.

5) $\int \frac{dx}{\sqrt{9x^2-6x+2}} = \int \frac{dx}{\sqrt{(3x-1)^2+1}} = \begin{bmatrix} t = 3x-1 \\ dt = 3dx \end{bmatrix} = \frac{1}{3}\int \frac{dt}{\sqrt{t^2+1^2}} = \frac{1}{3}\ln\left|t+\sqrt{t^2+1}\right|+C = \frac{1}{3}\ln\left|3x-1+\sqrt{9x^2-6x+2}\right|+C$;

6) $\int \frac{dx}{\sqrt{x^2-4x+5}} = \ln\left|x-2+\sqrt{x^2-4x+5}\right|+C$.

Kai

$$I_4 = \int \frac{Mx+N}{\sqrt{ax^2+bx+c}}dx$$

Pertvarkome analogiškai kaip I_2 ir gauname:

$I_4 = \frac{M}{2a}\int \frac{2ax+b}{\sqrt{ax^2+bx+c}}dx+\left(N-M\frac{b}{2a}\right)\int \frac{dx}{\sqrt{ax^2+bx+c}} = \frac{M}{a}\sqrt{ax^2+bx+c}+\left(N-M\frac{b}{2a}\right)I_3$.

$$7)\ \int\frac{2x-8}{\sqrt{1-x-x^2}}dx=-\int\frac{-2x-1-8-1}{\sqrt{1-x-x^2}}dx=-\int\frac{d\left(1-x-x^2\right)}{1-x-x^2}dx-9\int\frac{dx}{\sqrt{1-x-x^2}}=-2\sqrt{1-x-x^2}-9\int\frac{dx}{\sqrt{\frac{5}{4}-\left(x+\frac{1}{2}\right)^2}}=$$

$$=\begin{bmatrix}x+\frac{1}{2}=t\\ dx=dt\end{bmatrix}=-2\sqrt{1-x-x^2}-9\int\frac{dt}{\sqrt{\frac{5}{4}-t^2}}=-2\sqrt{1-x-x^2}-9\arcsin\frac{t}{\sqrt{5}/2}+C=-2\sqrt{1-x-x^2}-9\arcsin\frac{x+\frac{1}{2}}{\sqrt{5}/2}+C.$$

$$8)\ \int\frac{xdx}{\sqrt{5x^2-2x+1}}=\frac{1}{5}\sqrt{5x^2-2x+1}+\frac{1}{5\sqrt{5}}\ln\left|x\sqrt{5}-\frac{1}{\sqrt{5}}+\sqrt{5x^2-2x+1}\right|+C.$$

Kai

$$I_5=\int\frac{dx}{(Mx+N)\sqrt{ax^2+bx+c}}$$

pasirinkę keitinį $\frac{1}{t}=Mx+N,\ dx=-\frac{dt}{Mt^2}$ gausime I_3 tipo integralą.

$$9)\ \int\frac{dx}{(x+1)\sqrt{x^2+1}}=\begin{bmatrix}x+1=\frac{1}{t};\ x=\frac{1}{t}-1\\ t=\frac{1}{x+1}\\ dx=-\frac{dt}{t^2}\end{bmatrix}=\int\frac{-dt}{t^2\cdot\frac{1}{t}\sqrt{\left(\frac{1}{t}-1\right)^2+1}}=-\int\frac{dt}{t\frac{\sqrt{2t^2-2t+1}}{t}}=-\int\frac{dt}{\sqrt{2t^2-2t+1}}=-\frac{1}{\sqrt{2}}\int\frac{dt}{\sqrt{t^2-t+1/2}}=$$

$$=-\frac{1}{\sqrt{2}}\int\frac{dt}{\sqrt{\left(t-\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2}}=-\frac{1}{\sqrt{2}}\int\frac{d\left(t-\frac{1}{2}\right)}{\sqrt{\left(t-\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2}}=-\frac{1}{\sqrt{2}}\ln\left|\left(t-\frac{1}{2}\right)+\sqrt{\left(t-\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2}\right|+C=$$

$$=-\frac{1}{\sqrt{2}}\ln\left|\left(\frac{1}{x+1}-\frac{1}{2}\right)+\sqrt{\left(\frac{1}{x+1}-\frac{1}{2}\right)^2+\left(\frac{1}{4}\right)}\right|+C=C-\frac{1}{\sqrt{2}}\ln\left|\frac{1-x+\sqrt{2\left(x^2+1\right)}}{2(x+1)}\right|;$$

$$10)\ \int\frac{dx}{x\sqrt{x^2+x-1}}=C-\arcsin\frac{2-x}{x\sqrt{5}};$$

Kai

$$\int\frac{(Mx+N)}{(x-d)^k\sqrt{ax^2+bx+c}}dx,\ k=1,2,\ldots$$

parenkame keitinį $x-d=\frac{1}{t};\ dx=-\frac{dt}{t^2}$.

N.D.!!!

Apskaičiuokite integralus, kai pointegraliniame reiškinyje yra kvadratinis trinaris;

1. $\int\frac{dx}{x^2-7x+10}$	2. $\int\frac{dx}{x-x^2-2.5}$	3. $\int\frac{dx}{\sqrt{4x-3-x^2}}$	4. $\int\frac{dx}{\sqrt{1-(2x+3)^2}}$
5. $\int\frac{(x+2)dx}{x^2+2x+2}$	6. $\int\frac{(3x-1)dx}{4x^2-4x+17}$	7. $\int\frac{(x-3)dx}{\sqrt{3-2x-x^2}}$	8. $\int\frac{xdx}{2x^2-3x-2}$
9. $\int\frac{dx}{4x^2+4x+5}$	10. $\int\frac{xdx}{\sqrt{8+6x-9x^2}}$		